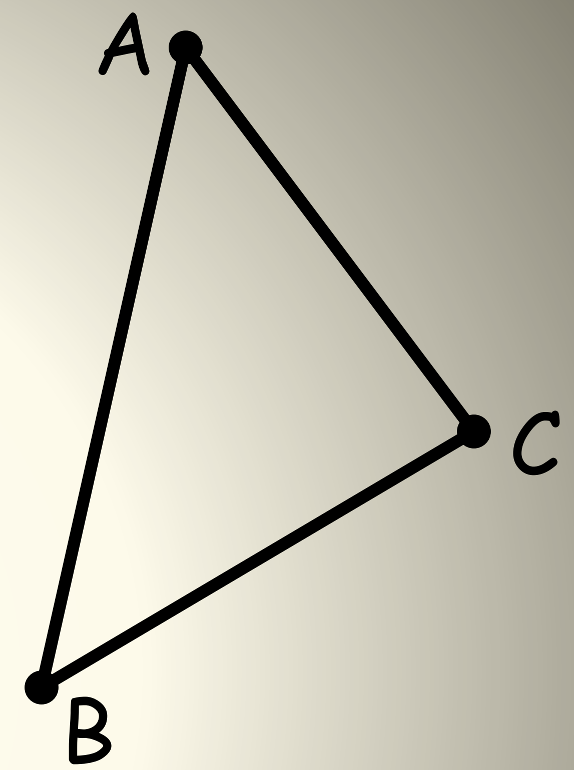
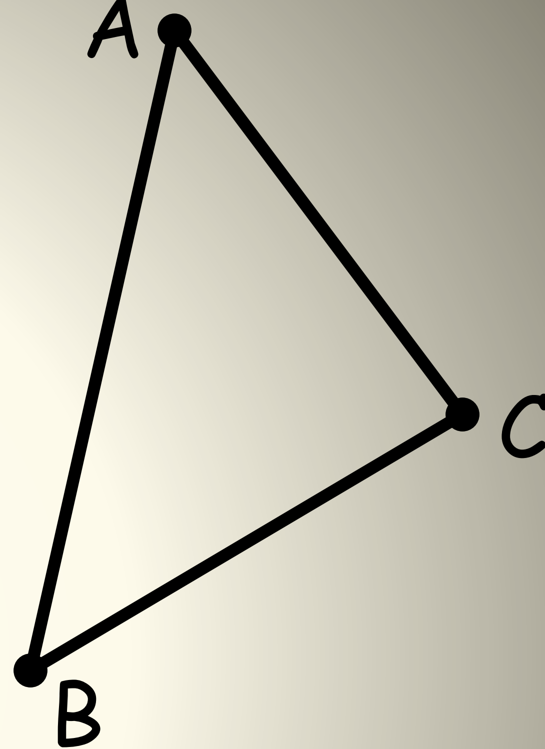
Section 3.4

Angles of a Triangle

A **Triangle** is the figure formed by three segments joining three noncollinear points.



Each of the points is a vertex of the triangle.



Example: $\triangle ABC$

Vertices: A, B, C

Angles: $\angle A, \angle B, \angle C$

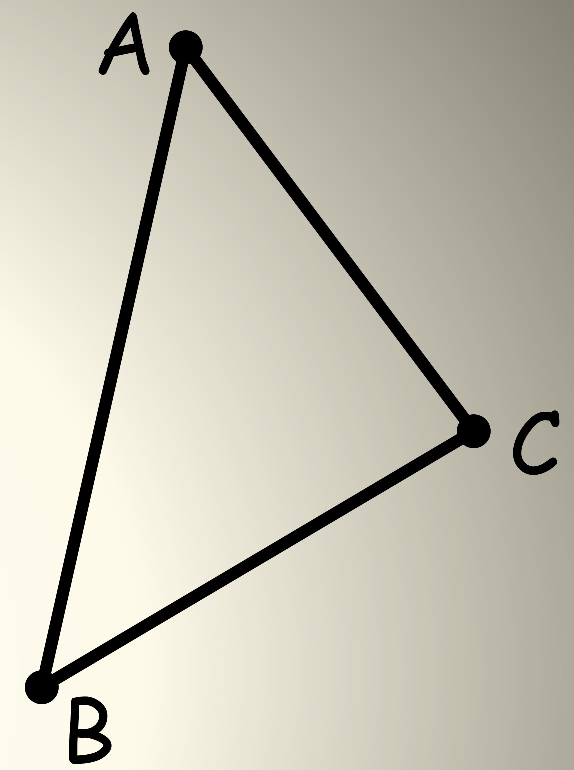
The segments are the sides of the triangle.

Sides: $\overline{AB}, \overline{BC}, \overline{AC}$

Side $\underline{\ \overline{BC}\ }$ is opposite $\angle A$

Side $\underline{\ \overline{AC}\ }$ is opposite $\angle B$

Side $\underline{\ \overline{AB}\ }$ is opposite $\angle C$



Side $\overline{AB}$ is included between $\angle$ $\underline{\ A\ }$ and $\angle$ $\underline{\ B\ }$

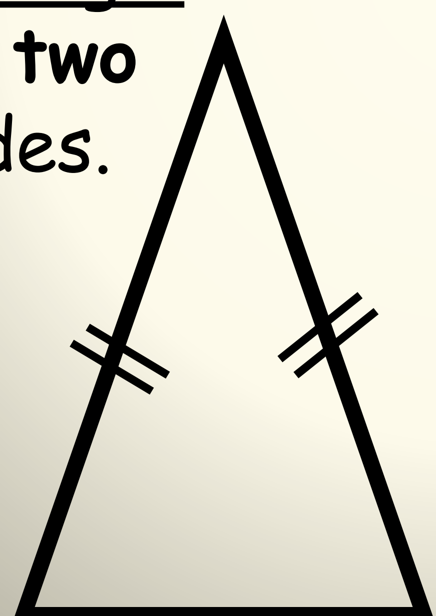
Side $\overline{AC}$ is included between $\angle$ $\underline{\ A\ }$ and $\angle$ $\underline{\ C\ }$

Side $\overline{BC}$ is included between $\angle$ $\underline{\ B\ }$ and $\angle$ $\underline{\ C\ }$

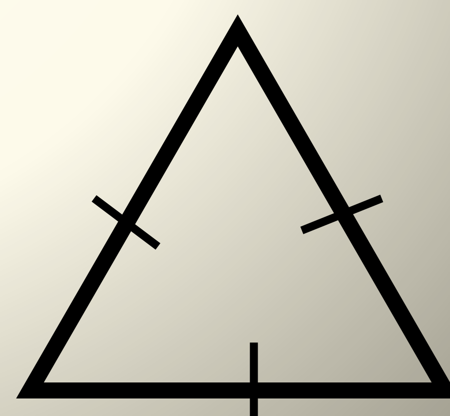
Classifying Triangles – By Number of Congruent Sides

Scalene Triangle – A triangle with **no** congruent sides.

Isosceles Triangle – A triangle with **two** congruent sides.

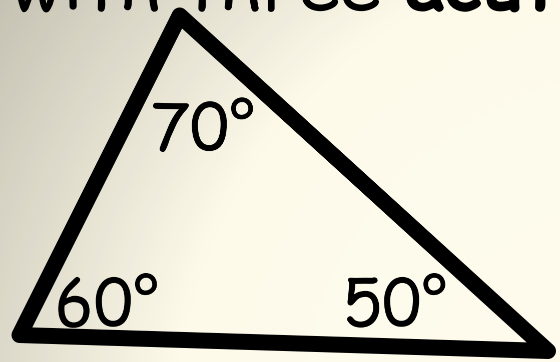


Equilateral Triangle – A triangle with **three** congruent sides.

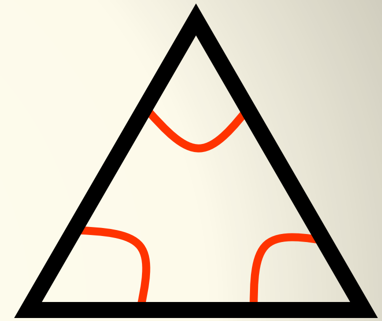


Classifying Triangles – By Angles

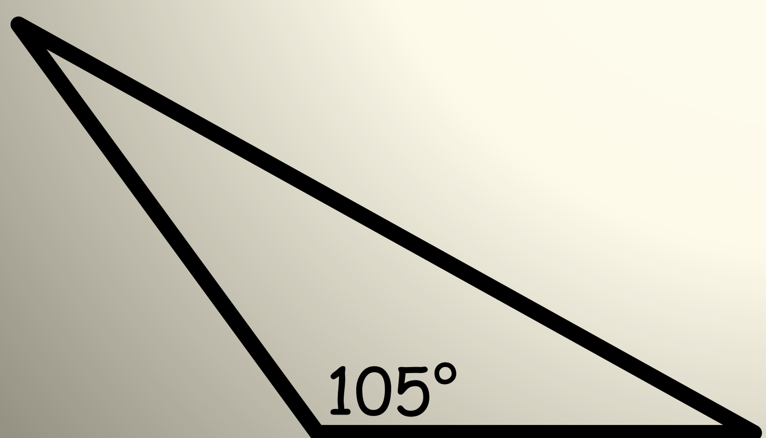
Acute Triangle – A triangle with three **acute angles**.



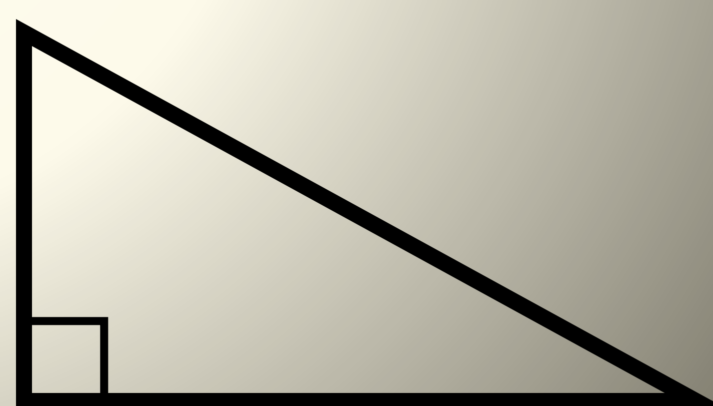
Equiangular Triangle – A triangle with three **congruent angles**.



Obtuse Triangle – A triangle with one **obtuse** angle.

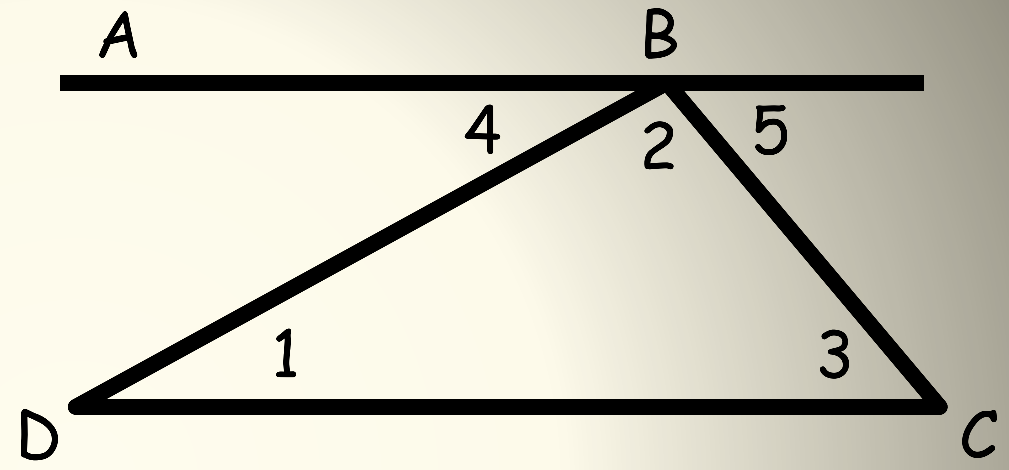


Right Triangle – A triangle with one **right** angle.



Theorem 3-11: the sum of the angles of a triangle is 180.

Auxiliary Line
- A line added to a diagram to help in a proof

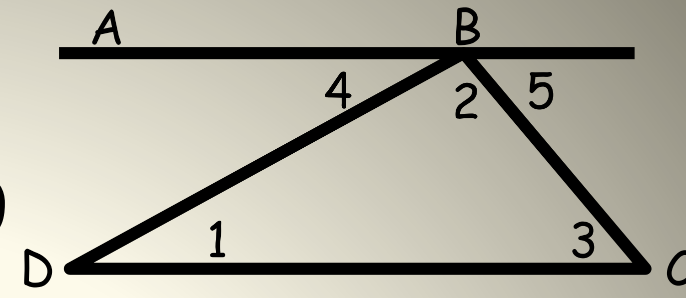


Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$

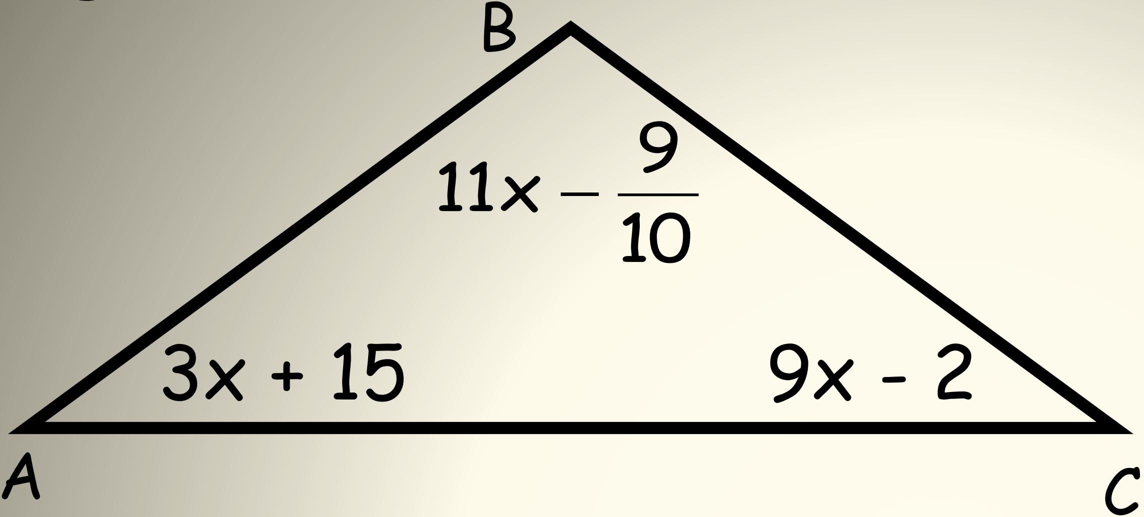
Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$



Statements	Reasons
1. $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$	1. Given
2. $m\angle 1 = m\angle 4$; $m\angle 3 = m\angle 5$	2. If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
3. $m\angle ABC = m\angle 4 + m\angle 2$	3. Angle Addition Postulate
4. $m\angle ABC + m\angle 5 = 180.$	4. Angle Addition Postulate
5. $m\angle 4 + m\angle 2 + m\angle 5 = 180$	5. Substitution
6. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	6. Substitution

Algebra Connection



$$3x + 15 + 9x - 2 + 11x - \frac{9}{10} = 180$$

$$23x + 12.1 = 180$$

$$23x = 167.9$$

$$\boxed{x = 7.3}$$

$m\angle A = 3(7.3) + 15$

$m\angle A = 36.9$

$m\angle C = 9(7.3) - 2$

$m\angle C = 63.7$

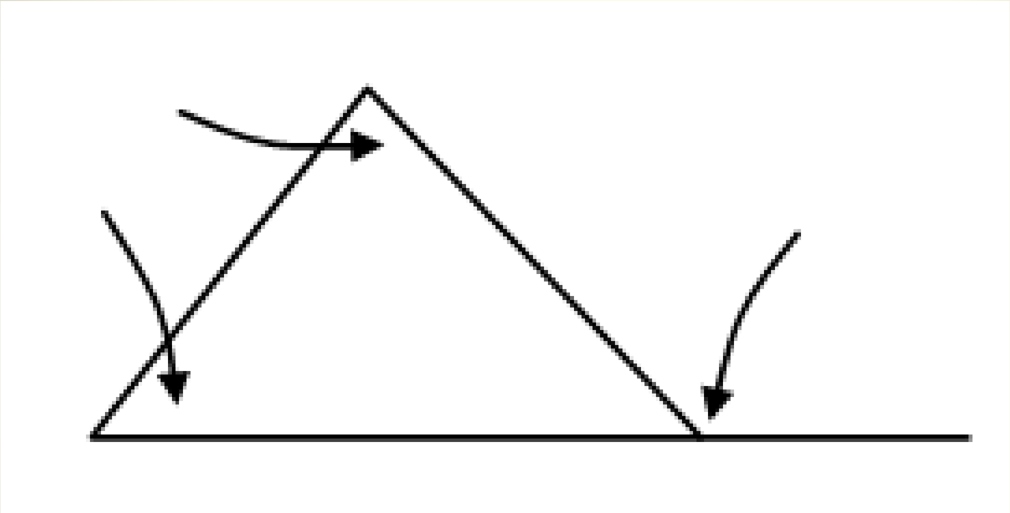
$m\angle B = 11(7.3) - \frac{9}{10}$

$m\angle B = 79.4$

Check:

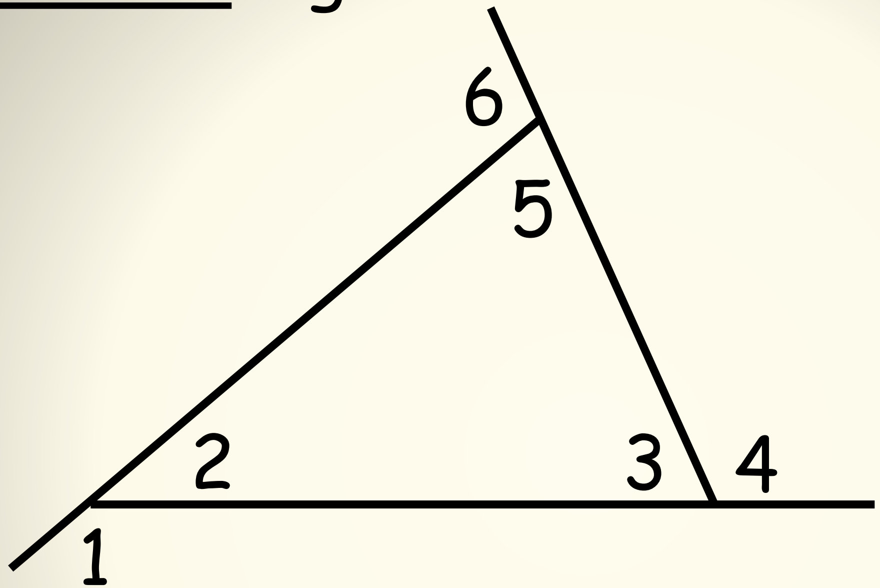
$36.9 + 63.7 + 79.4 = \underline{\ \ 180\ \ }$

An **exterior** angle if formed when one side of a triangle is extended.



The **remote interior** angles are two angles of a triangle not adjacent to the exterior angle.

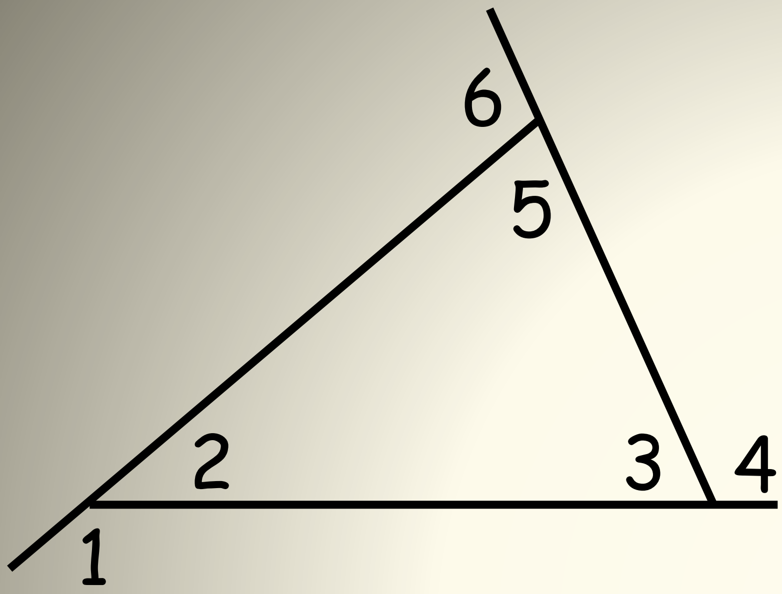
Theorem 3-12: The measure of an **exterior** angle of a triangle equals the sum of the measures of the two **remote interior** angles.



$m\angle 3 + m\angle 5 = m\angle$ 1

$m\angle 2 + m\angle 3 = m\angle$ 6

$m\angle 2 + m\angle 5 = m\angle$ 4



$m\angle 1 = 135^\circ$

$m\angle 2 = 45^\circ$

$m\angle 3 = 60^\circ$

$m\angle 4 = 120^\circ$

$m\angle 5 = 75^\circ$

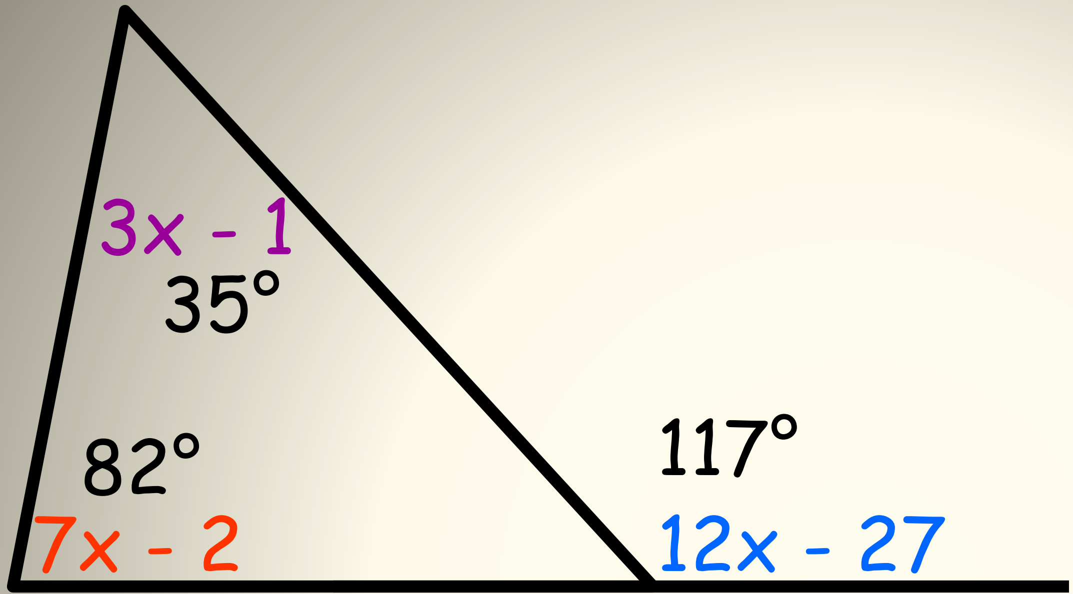
$m\angle 6 = 105^\circ$

$m\angle 3 + m\angle 5 = m\angle$ 1

$m\angle 2 + m\angle 3 = m\angle$ 6

$m\angle 2 + m\angle 5 = m\angle$ 4

Algebra Connection



$$3x - 1 + 7x - 2 = 12x - 27$$

$$10x - 3 = 12x - 27$$

$$24 = 2x$$

$$\boxed{12 = x}$$

w↑

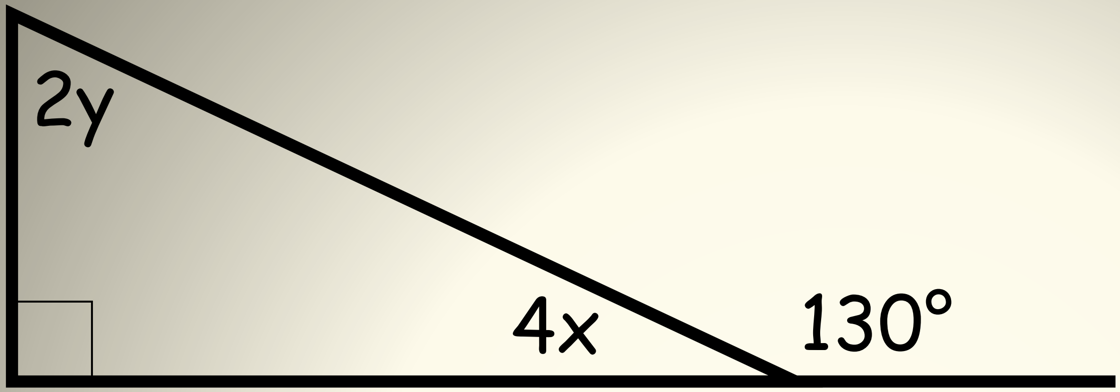
- What is the name for an extra line drawn to aid in a proof?
- Lines that are on opposite sides of the transversal contained within two lines that are cut by a transversal are called _______________ .
- My favorite TV show is _______________ .

Homework Check – page 97

- Numbers 1-4: Volunteers to draw on board?

5. 180
6. 30
7. 95
8. 50
9. 25
10. 360
11. x = 30, y = 80
12. x = 110, y = 70
13. x = 40, y = 50

Algebra Connection



$2y + 90 = 130$

$2y = 40$

$\boxed{y = 20}$

$4x + 130 = 180$

$4x = 50$

$\boxed{x = 12.5}$

Vocabulary

- **Postulate** – a statement that is accepted without proof
- **Theorem** – a statement that can be proved
- **Corollary** – a statement that can be proved by applying a theorem is a corollary of that theorem

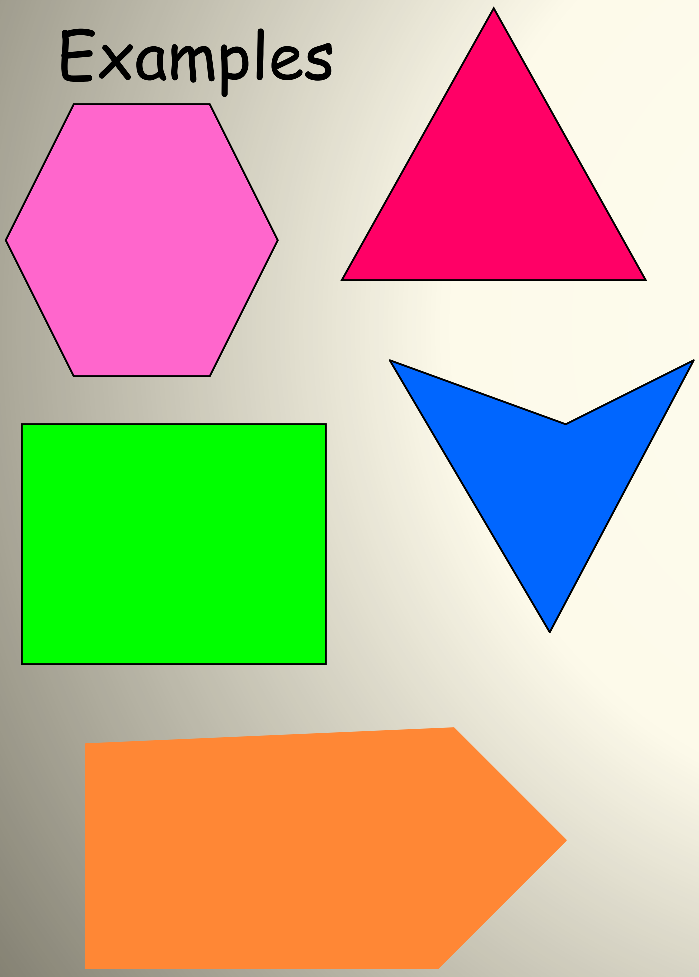
Corollaries for this Section

- **Corollary 1**
  - If the two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent
- **Corollary 2**
  - Each angle of an equiangular triangle has a measure of 60 degrees
- **Corollary 3**
  - In a triangle, there can be at most one right angle or one obtuse angle
- **Corollary 4**
  - The acute angles of a right triangle are complementary

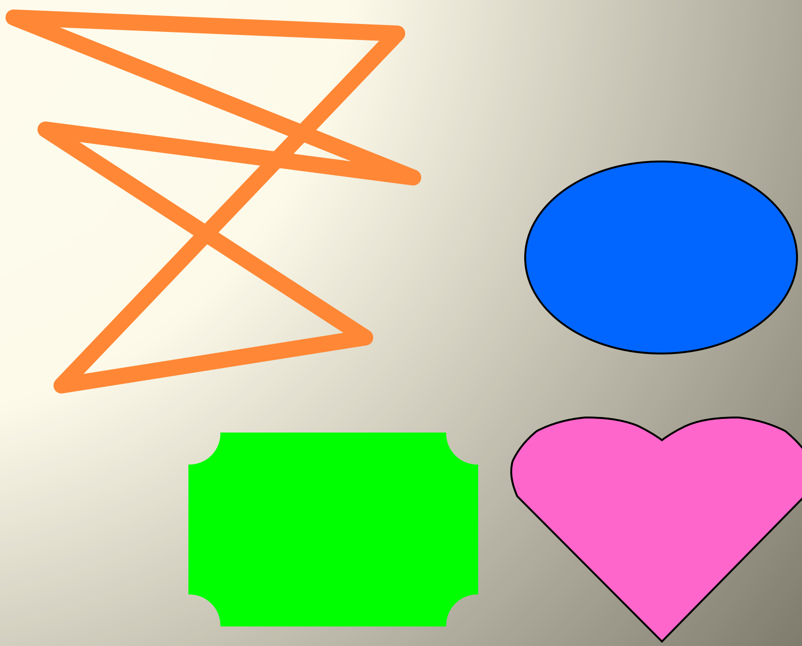
Polygons

Polygon means "Many Angles"
- Closed figure, and sides are line segments (straight)

Examples

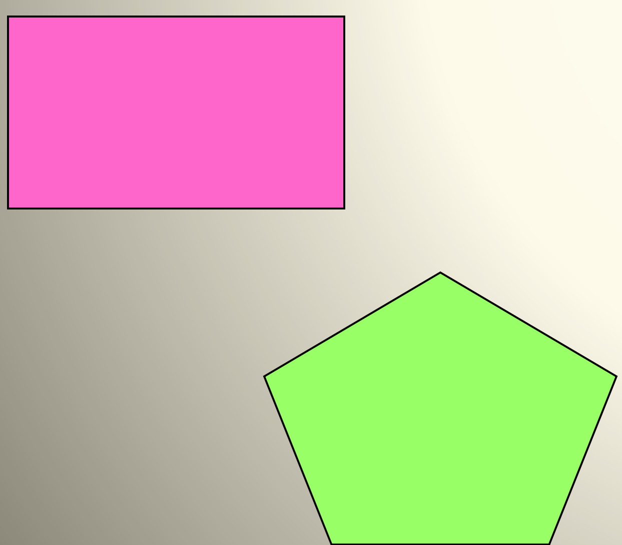


Not Polygons

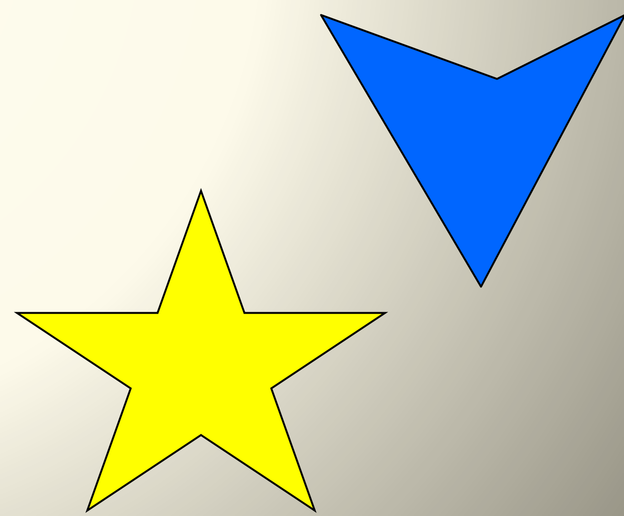


Convex Polygon – any polygon such that no line segment can be drawn between two vertices on the exterior of the polygon.

Convex

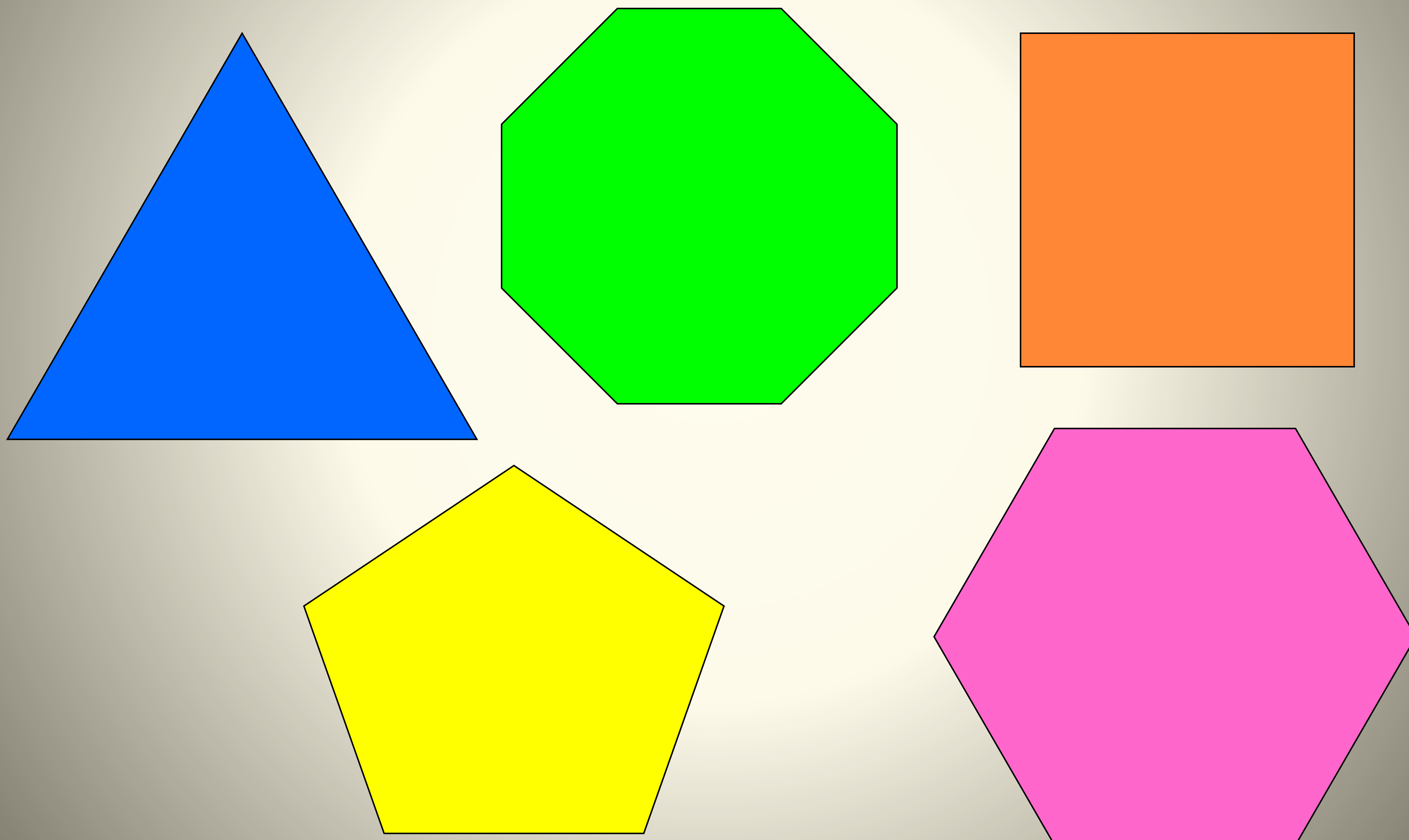


Not Convex

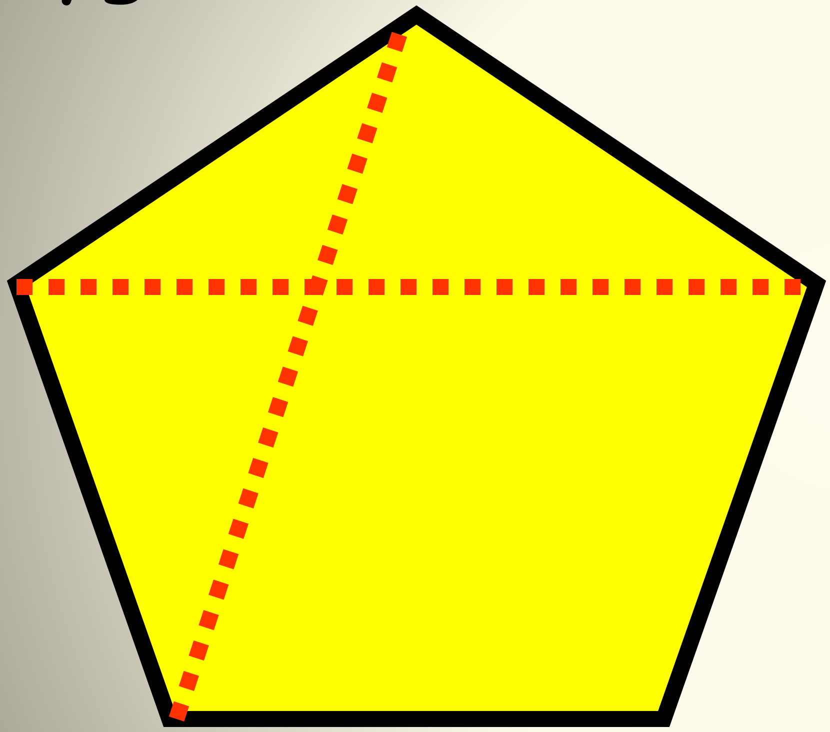


Regular Polygon – a polygon that is both equilateral and equiangular.

*Notation on figure***



<u>Diagonal</u> – a segment joining two nonconsecutive vertices of a convex polygon.



Theorem 3-13:

The sum of the measures of the interior angles of a convex polygon with n sides is

$$(n-2)*180$$

Theorem 3-14:

The sum of the measures of the exterior angles of any convex polygon, one angle at each vertex is 360°

Number of Sides	Name	Sum of Interior Angles	Measure of each angle (if the polygon is regular)
3	Triangle	180°	60°
4	Quadrilateral	360°	90°
5	Pentagon	540°	108°
6	Hexagon	720°	120°
7	Heptagon	900°	128.57°
8	Octagon	1080°	135°
9	Nonagon	1260°	140°
10	Decagon	1440°	144°
n	n-gon	$(n - 2)\times 180°$	$\frac{(n-2)\times 180}{n}$

Number of Sides	Sum of Exterior Angles	Measure of EACH exterior angle (if the polygon is regular)
3	360°	120°
4	360°	90°
5	360°	72°
6	360°	60°
7	360°	51.43°
8	360°	45°
9	360°	40°
10	360°	36°
n	360°	$\frac{360}{n}$